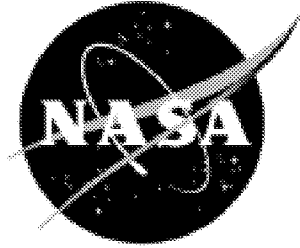


NASA/CR-2002-211635



Design of a Dual Waveguide Normal Incidence Tube (DWNIT) Utilizing Energy and Modal Methods

Juan F. Betts

Lockheed Martin Corporation, Hampton, Virginia

April 2002

The NASA STI Program Office . . . in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- **TECHNICAL PUBLICATION.** Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counterpart of peer-reviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.
- **TECHNICAL MEMORANDUM.** Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.
- **CONTRACTOR REPORT.** Scientific and technical findings by NASA-sponsored contractors and grantees.

- **CONFERENCE PUBLICATION.** Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.
- **SPECIAL PUBLICATION.** Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.

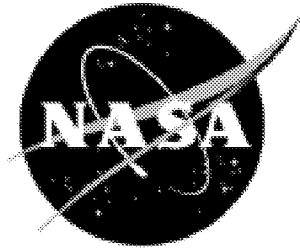
TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results . . . even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at <http://www.sti.nasa.gov>
- Email your question via the Internet to help@sti.nasa.gov
- Fax your question to the NASA STI Help Desk at (301) 621-0134
- Telephone the NASA STI Help Desk at (301) 621-0390
- Write to:
NASA STI Help Desk
NASA Center for AeroSpace Information
7121 Standard Drive
Hanover, MD 21076-1320

NASA/CR-2002-211635



Design of a Dual Waveguide Normal Incidence Tube (DWNIT) Utilizing Energy and Modal Methods

Juan F. Betts

Lockheed Martin Corporation, Hampton, Virginia

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23681-2199

Prepared for Langley Research Center
under Contract NAS1-00135

April 2002

Acknowledgments

The author would like to extend a special thanks to Mr. Tony Parrott and Mr. Michael Jones from the Structural Acoustic Branch at NASA Langley Research Center for their guidance and support of this effort. The Dual Waveguide Normal Incidence (DWNIT) design concept presented in this report was developed by them, and this report presents a design analysis based on their design concept. The author also appreciates Mr. Parrott's gracious permission in allowing the use of his design figures for this report. I would also like to thank Mr. Brian Howerton from Lockheed Martin Space Operations for his experimental insights on this design analysis. Finally, I would like to thank Dr. Jesse Follet from The Boeing Company for graciously allowing the use of his picture schematics of the current Normal Incidence Tube (NIT).

Available from:

NASA Center for AeroSpace Information (CASI)
7121 Standard Drive
Hanover, MD 21076-1320
(301) 621-0390

National Technical Information Service (NTIS)
5285 Port Royal Road
Springfield, VA 22161-2171
(703) 605-6000

Abstract

This report presents an analysis on the partition design of the proposed Dual Waveguide Normal Incidence Tube (DWNIT). Some of the proposed usages of the DWNIT are:

1. Assessment of coupling relationships between resonators in close proximity
2. Evaluation of "smart liners"
3. Experimental validation for parallel element models
4. Investigation of effects of simulated angles of incidence through the introduction of higher order cross-sectional acoustic modes

To address this design challenge, an energy model was developed of the two chambers. This energy model was used to determine the SPL drop across the two chambers, through the use of an intensity transmission function for the chamber's partition. The model also allowed the chamber's lengthwise end samples to vary.

To model the partition, two intensity transmission function models were studied. The first intensity transfer function model used was a simple compressive model and the other was a pseudo-plate bending model.

The initial partition design (2" high, 16" long, 0.25" thick) was predicted to provide at least 160 dB SPL drop across the partition with the compressive model, and at least 240 dB SPL drop with the bending model. These results were produced when the structural damping loss in the bending model was 0.01 and the model end chamber sample transmissions were set to 0.1. This was done to analyze the partition under a close to worst-case scenario.

Since these results predicted more SPL drop than required, a plate thickness optimization algorithm was developed. The results of the algorithm routine indicated that a plate with the same height and length, but with a thickness of 0.1" and 0.05 structural damping loss, would provide an adequate SPL isolation between the chambers. This partition design would provide 210 dB and 125 dB SPL drop across the partition, utilizing the compressive and bending models, respectively. These results were also

computed with the end chamber sample transmissions set to 0.1. When more realistic end intensity transmission coefficients (e.g., 0.555) were used, the compressive and bending models predicted a partition isolation of 250 dB and 160 dB, respectively.

Table of Contents

1. Introduction	1
1.1 Objective	1
1.2 Approach	4
2. Theoretical Background	5
2.1 Energy Model Development	5
2.2 Chamber Coupling Intensity Transfer Function.....	9
2.2.1 Compressive Model.....	9
2.2.2 Pseudo-Plate Model (Bending Model)	10
2.3 Partition Thickness and Structural Damping Loss Factor.....	16
3. Results	18
4. Conclusions	28

List of Figures and Tables

Figure 1-1. Schematic of Current NIT (perspective view).¹

Figure 1-2. Schematic of Current NIT (side view).¹

Figure 1-3. Schematic of Partition Design for the Proposed DWNIT. (isometric view)²

Figure 1-4. Schematic of Partition Design for the Proposed DWNIT (side view).²

Figure 2-1. DWNIT model schematic.

Figure 2-2. Compressive model schematic.

Figure 2-3. Bending model schematic.

Figure 2-4. End taper design of plate partition.²

Figure 3-1. Intensity Transfer Function for a 0.25in (0.635cm) thick partition utilizing the Compressive Model.

Figure 3-2. SPL drop across a 0.25in (0.635cm) thick partition utilizing the Compressive Model with $T_{1e}=T_{2e}=0.1$

Figure 3-3. Intensity Transfer Function for a 0.25in (0.635cm) thick partition utilizing the Bending Model.

Figure 3-4. SPL drop across a 0.25in (0.635cm) thick partition utilizing the Bending Model with $T_{1e}=T_{2e}=0.1$

Figure 3-5. Dispersion relationship utilizing a 0.25in (0.635cm) thick partition.

Figure 3-6. Minimum SPL Drop vs. Partition Thickness Utilizing the Bending Model.

Figure 3-7. Minimum SPL Drop vs. Partition Thickness Utilizing the Compressive Model.

Figure 3-8. SPL drop across a 0.1in (0.254cm) thick partition utilizing the Bending Model with $T_{1e}=T_{2e}=0.1$

Figure 3-9. SPL drop across a 0.1in (0.254cm) thick partition utilizing the Bending Model with $T_{1e}=T_{2e}=0.555$

Figure 3-10. SPL drop across a 0.1in (0.254cm) thick partition utilizing the Compressive Model with $T_{1e}=T_{2e}=0.555$.

¹ Reprinted with permission of Dr. Jesse Follet

² Reprinted with permission of Mr. Tony Parrott

List of Variables and Abbreviations

a	length of the of the partition plate
a_e	effective length of the of the partition plate
A_n	modal amplitude n
b	height of the partition plate
c	speed of sound in air
cm	centimeters
dB	decibel
dB_Drop	sound pressure level drop across the plate partition
D	beam bending stiffness
DWNIT	Dual Waveguide Normal Incidence Tube
E	plate modulus of elasticity
f	frequency
in	inches
Hz	hertz
I	sound intensity
I_b	moment of inertia of the plate
I_{ref}	reference impedance
I_{12in}	Sound intensity incident on surface 12
I_{12ref}	Sound intensity reflected from surface 12
I_{12out}	Sound intensity transmitted into surface 12
I_{1ein}	Sound intensity incident on surface 1e
I_{1eref}	Sound intensity reflected from surface 1e
I_{1eout}	Sound intensity transmitted into surface 1e
I_{2ein}	Sound intensity incident on surface 2e
I_{2eref}	Sound intensity reflected from surface 2e
I_{2eout}	Sound intensity transmitted into surface 2e
k	free acoustic wave number
k_1	wave number in the axial direction (x direction)
k_2	wave number in the normal direction (y direction)

k_b	bending plate wave number
k_{be}	effective bending plate wave number
kHz	kilohertz
m	mass per unit length of beam
Max	the Max function (finds the minimum of a given function)
Min	the Min function (finds the minimum of a given function)
NIT	Normal Incidence Tube
p.d.e.	partial differential equation
P_{in}	incident acoustic wave on the partition
P_{out}	transmitted acoustic wave from the partition
q	total number of bending modes used in model
r_{air}	characteristic impedance of air (equals ρc)
r_{metal}	characteristic impedance of metal used for the partition
R.H.S.	right hand side
RMS	root mean square
S	equals $S_{12}+S_{1e}$
S_{12}	partition's surface area
S_{1e}	surface area of surface 1e
S_{2e}	surface area of surface 2e
SIL	sound intensity level
SIL_1	sound intensity level in chamber 1 of the DWNIT
SIL_2	sound intensity level in chamber 2 of the DWNIT
SPL	sound pressure level
ΔSPL	sound pressure level drop across partition plate
t	time
T	energy transmission coefficient or transfer function
T_{12}	Intensity transfer function for surface 12
T_{1e}	Intensity transfer function for surface 1e
T_{2e}	Intensity transfer function for surface 2e
W	acoustic power
x	axis along the length of the plate partition

y	axis normal to the plate partition
Z	acoustic impedance
χ	ρc normalized nondimensional reactance
δ_m	Kronica delta function (equals 1 when $r=n$ and zero when $r \neq n$)
ϕ_n	mode shape n
γ	structural damping loss factor
$\eta(x,t)$	plate partition displacement
θ	ρc normalized nondimensional resistance
ρ	density of air at room temperature
τ	thickness of the plate partition
ω	circular frequency (equals $2\pi f$)
ω_n	natural circular frequency
Ψ_n	equals $\int_0^a \phi_n(x) dx$

Throughout this report both English and SI units are used. English values and units are followed by the equivalent SI values and units in parentheses. The report follows an $e^{+i\omega t}$ convention unless otherwise indicated. The symbol $*$ indicates a complex conjugate quantity. Quantities enclosed by $| \quad |$ indicate absolute values. Throughout this report, terms such as transfer function and transmission coefficient are used interchangeably, and indicate the ratio of output vs. input intensity in the frequency domain. References are located at the end of each chapter for the reader's convenience.

1. Introduction

1.1 Objective

The objective of this report is to analyze the partition design for the proposed NASA Langley Dual Waveguide Normal Incident Tube (DWNIT). The DWNIT allows simultaneous measurements of two end sample impedances. The proposed design adds a partition lengthwise to the current NASA Langley Normal Incident Tube (NIT).

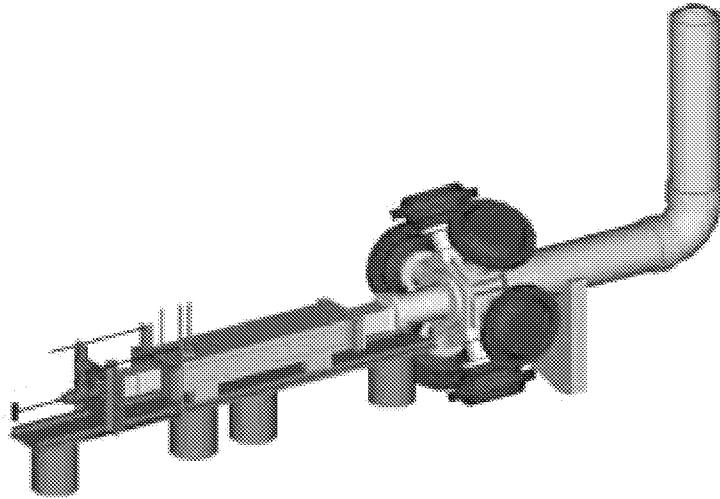


Figure 1-1. Schematic of Current NIT.* (perspective view)

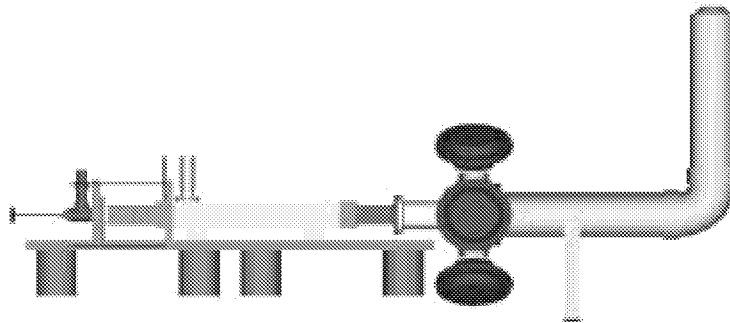


Figure 1-2. Schematic of Current NIT.* (side view)

* Reprinted with permission of Dr. Jesse Follet

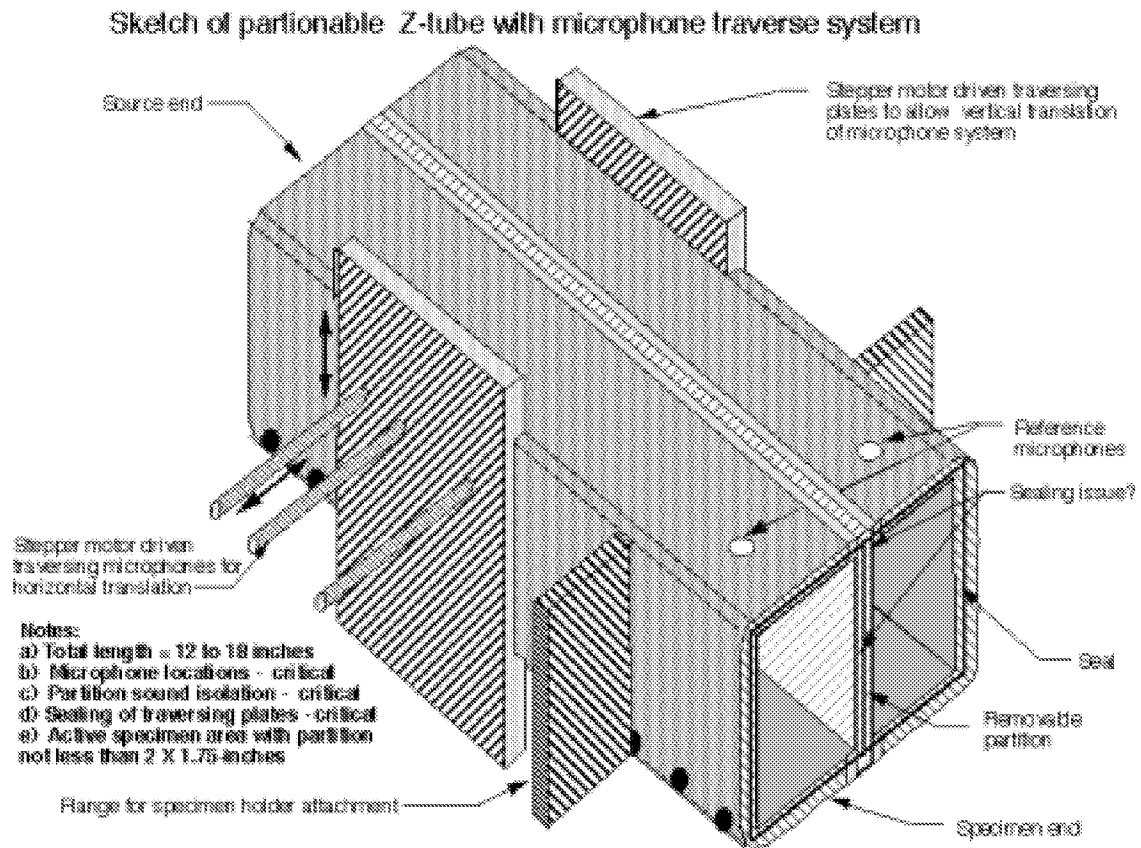


Figure 1-3. Schematic of Partition Design for the Proposed DWNIT (isometric view).[†]

[†] Reprinted with permission of Mr. Tony Parrott

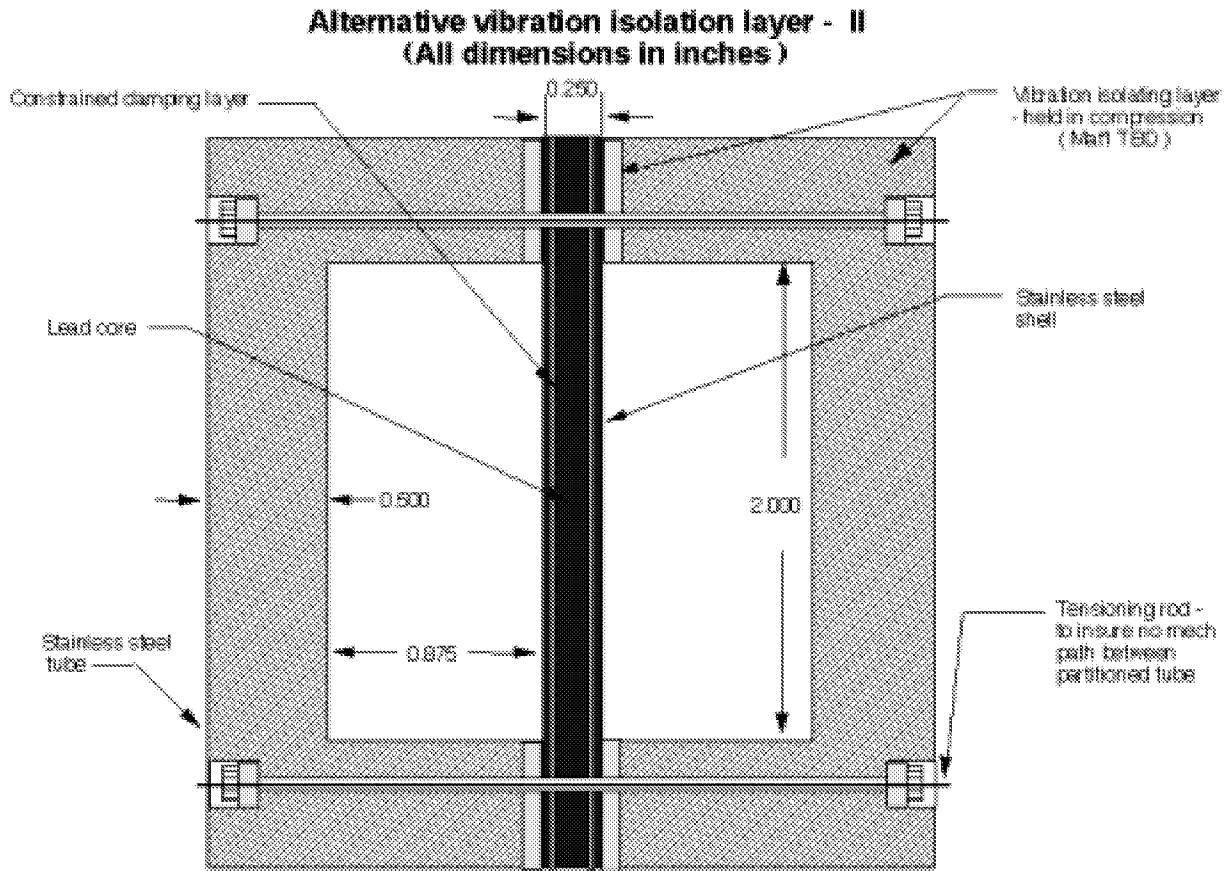


Figure 1-4. Schematic of Partition Design for the Proposed DWNIT (side view).[†]

Since the purpose of the DWNIT is to provide simultaneous and independent assessment of the impedances of two samples, the partition should be as thin as possible while providing at least 40 dB sound pressure level (SPL) isolation between each of its lengthwise chambers. Figures 1-1 and 1-2 show the current NASA Langley NIT and Figs. 1-3 and 1-4 show the proposed partition design for the DWNIT.

Some of the advantages, in the study of advanced liner concepts, provided by the DWNIT are:

1. Assessment of coupling relationships between resonators in close proximity
2. Evaluation of "smart liners"
3. Experimental validation for parallel element models
4. Investigation of effects of simulated angles of incidence through the introduction of higher order cross-sectional acoustic modes

1.2 Approach

There are several design challenges associated with the DWNIT. This report will address one of the major challenges associated with the design of the DWNIT, specifically the design of the two-chamber partition. This partition must not allow acoustic energy from one chamber to leak into the other chamber and therefore distort the acoustic impedance of the end sample in the other chamber. The design objective is for the partition to provide at least 40 dB sound pressure level (SPL) drop isolation between the two chambers.

To address this design challenge, an energy model was developed of the two chambers. This energy model determined the SPL drop across the two chambers using an intensity transmission function for the chamber's partition. The model also allowed for the samples at the end of the two NIT chambers to vary.

To model the partition, two intensity transfer function models were studied. The first intensity transfer function model used was a simple compressive model.¹ This model assumes some of the incident acoustic wave is transmitted and the remaining portion is reflected. The impedance of the partition is simply associated with the interaction of the characteristic impedance (ρc) of air and that of the material associated with the partition. The solution of this model is provided by Kinsler et al in Ref. 1

The second intensity transfer function model was a structural acoustic interaction model.² This model assumed the partition would transmit sound from one chamber to the other through bending waves generated in the partition by the incident acoustic field. This model took into account the bending modes associated with a simply supported plate. The model also includes the radiation coincidence effect associated with the dispersion relationship.

References

1. Kinsler, L.E., Frey, A.R., Coppens, A.B., and Sanders, J.V., *Fundamental of Acoustics*, John Wiley & Sons, New York, 1982.
2. Fahy, F., *Sound Structural Vibration*, Academic Press, New York, 1998.

2. Theoretical Background

2.1 Energy Model Development

Figure 2-1 shows a schematic of the two chamber of the DWNIT. Variables T_{1e} , T_{2e} and T_{12} are the intensity transmission functions of the end sample in chamber one, the end sample in chamber two, and the partition plate between chambers one and two, respectively. T_{1e} and T_{2e} are arbitrarily changed to study their variation effects on the SPL drop across the partition, while T_{12} is set by the transmission models developed in Section 2.2.

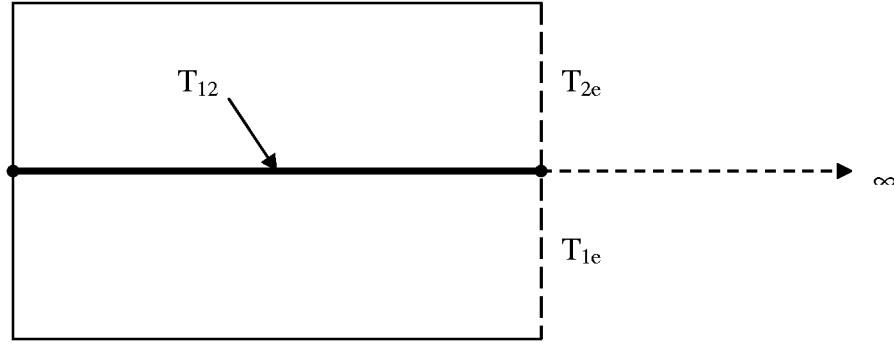


Figure 2-1. DWNIT model schematic.

To develop the energy model, intensity boundary conditions were applied to each of the three energy transmission surfaces. The intensity boundary conditions for surfaces T_{12} , T_{2e} , and T_{1e} are given by:

$$\begin{aligned} I_{12out} &= I_{12in} T_{12} \\ I_{12ref} &= (1 - T_{12}) I_{12in} \end{aligned} \quad (2-1)$$

$$\begin{aligned}
I_{2eout} &= T_{2e} I_{2ein} \\
(I_{2ein} - I_{2eref}) S_{2e} &= S_{12} I_{12out} \\
I_{2eref} &= (1 - T_{2e}) I_{2ein}
\end{aligned} \tag{2-2}$$

$$\begin{aligned}
I_{1eout} &= I_{1ein} T_{1e} \\
I_{1eref} &= (1 - T_{1e}) I_{1ein}
\end{aligned} \tag{2-3}$$

respectively.

The acoustic intensity (I) is related to the acoustic power (W) through

$$\begin{aligned}
I &= \frac{dW}{dS} \\
W &= \int I dS
\end{aligned} \tag{2-4}$$

where S is the total area. If the intensity is assumed to be constant through the surfaces, then $W=IS$. For this model it is assumed that there is an acoustic power in chamber one and no power source in chamber two. Therefore the total power input is

$$W = (I_{12in} - I_{12ref}) S_{12} + (I_{1ein} - I_{1eref}) S_{1e} \tag{2-5}$$

The total power I_{12in} and I_{1ein} can be related by:

$$\begin{aligned}
I_{12in} &= \frac{T_{12}}{T_{tot}} \frac{W}{S} \\
I_{1ein} &= \frac{T_{1e}}{T_{tot}} \frac{W}{S}
\end{aligned} \tag{2-6}$$

Therefore from Eq. (2-6)

$$\frac{I_{12}}{I_{1e}} = \frac{T_{12}}{T_{1e}} \tag{2-7}$$

For plane waves, the intensity is related to the RMS acoustic pressure (P_{rms}) by

$$I = \frac{P_{rms}^2}{\rho c}$$

$$\therefore W = IS = \frac{P_{rms}^2}{\rho c} S \quad (2-8)$$

The SPL and Sound Intensity Level (SIL) are given by

$$SIL = 10 \log \left\{ \frac{I}{I_{ref.}} \right\} \quad (2-9)$$

$$SPL = 10 \log \left\{ \frac{P_{rms}^2}{P_{ref}^2} \right\}$$

and for plane waves

$$SIL \approx SPL \quad (2-10)$$

For incoherent sources

$$P_{rmstotal}^2 = P_{rms1}^2 + P_{rms2}^2 + \dots + P_{rmsn}^2 \quad (2-11)$$

To determine the SPL drop across the partition, the following procedure is followed. From Eqs. (2-1) through (2-3), the total intensity in chambers one (I_1) and two (I_2) can be determined. The SIL for each chamber can be used instead of the SPL, and subtracting SIL_1 from SIL_2 will give the SPL drop across the partition.

Therefore, I_1 is given by

$$I_1 = I_{12in} + I_{12ref} + I_{1ein} + I_{1eref} \quad (2-12)$$

Using Eqs. (2-1), (2-3), (2-7), and (2-12) produces

$$I_1 = \left[(2 - T_{12}) + (2 - T_{1e}) \right] \frac{T_{1e}}{T_{12}} I_{12in} \quad (2-13)$$

I_2 is given by

$$I_2 = I_{2ein} + I_{2eref} \quad (2-14)$$

using Eqs. (2-2) and (2-14) produces

$$I_2 = (2 - T_{2e}) \frac{S_{12}}{S_{2e}} \frac{T_{12}}{T_{2e}} I_{12in} \quad (2-15)$$

The SPL drop across the partition is

$$\Delta SPL \approx SIL_1 - SIL_2 = 10 \log \left[\frac{I_1}{I_{ref}} \right] - 10 \log \left[\frac{I_2}{I_{ref}} \right] = 10 \log \left[\frac{I_1}{I_2} \right] \quad (2-16)$$

Therefore ΔSPL is

$$\Delta SPL \approx 10 \log \left[\frac{\left[(2 - T_{12}) + (2 - T_{1e}) \right] \frac{T_{1e}}{T_{12}}}{(2 - T_{2e}) \frac{S_{12}}{S_{2e}} \frac{T_{12}}{T_{2e}}} \right] \quad (2-17)$$

2.2 Chamber Coupling Intensity Transfer Function

2.2.1 Compressive Model

The energy model developed in Section 2.1 required as an input the intensity transmission function of the partition (T_{12}). The simplest model for this transfer function is a compressive acoustic waves incident on an infinitely long metal as shown in Fig. 2-2.

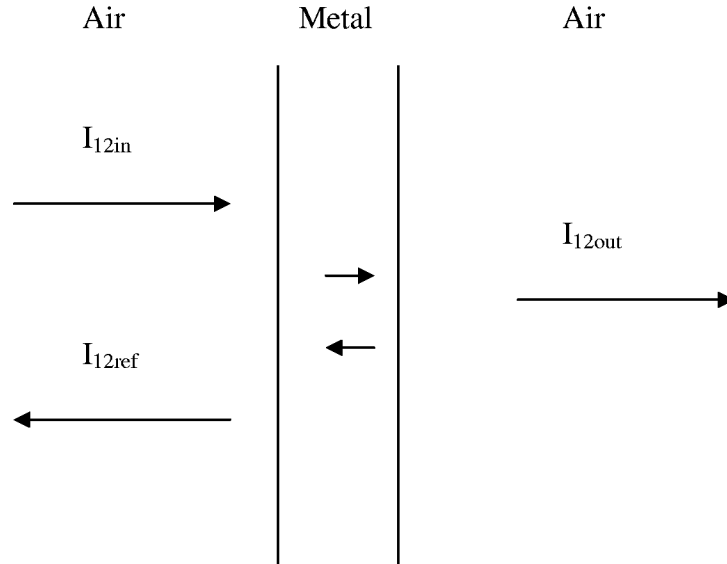


Figure 2-2. Compressive model schematic.

The solution for the transmission function T_{12} is given by Kinsler et al¹ as

$$T_{12} = \left| \frac{2r_{air}}{r_{metal} \sin(k_{metal} \tau)} \right| \quad (2-18)$$

where r_{air} and r_{metal} are the characteristic impedance (pc) of air and the metal used, respectively, and k_{metal} and τ are the wave number and thickness of the partition metal, respectively.

2.2.2 Pseudo-Plate Model (Bending Model)

In order to better assess the energy transmitted from one chamber of the DWNIT to the other, a model that accounts for the structural bending response of the partition was needed. A simply supported pseudo-plate bending model for the chamber's partition is presented in this section. Figure 2-3 shows a schematic of the model used in this section.

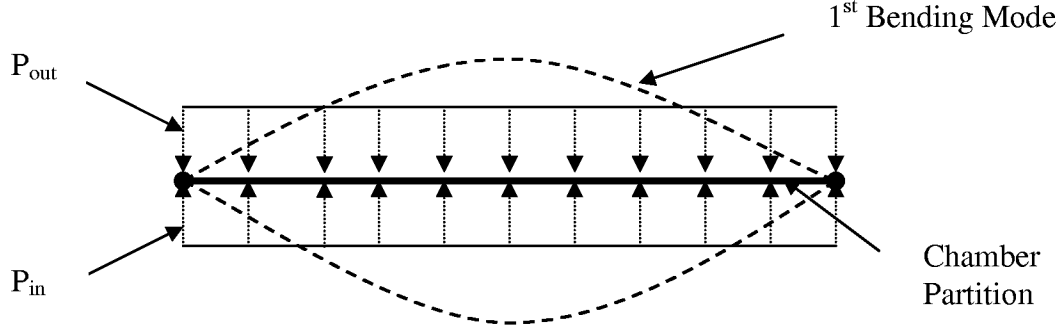


Figure 2-3. Bending model schematic.

The sound structure interaction under various conditions has been studied by a number of authors. The analysis that follows uses some of the analysis elements of Fahy.² The governing differential equation of a beam is given by:

$$D \frac{\partial^4 \eta(x, t)}{\partial x^4} + m \frac{\partial^2 \eta(x, t)}{\partial t^2} = (P_{in} - P_{out}) b e^{i\omega t} \quad (2-19)$$

where D and m are the dynamic stiffness and mass per unit length, respectively, of the beam. For a beam, D is given by $EI_b(1+i\gamma)$, where E is the modulus of elasticity, I_b is the moment of inertia, and γ is the structural damping loss factor. P_{in} and P_{out} are the incident and transmitted acoustic pressures, respectively. The solution of Eq. (2-19) can be written as a summation of modal amplitudes multiplied by their mode shapes, which takes the following form:

$$\eta(x, t) = \sum_{n=1}^q A_n(\omega) \phi_n(x) e^{i\omega t} = A_n \phi_n e^{i\omega t} \quad (2-20)$$

where the summation symbol is eliminated for simplicity and the Einstein's summation convention is used, where repeated indices imply summation. ϕ_n is the *in vacuo* mode shape for a given set of boundary conditions. The mode shapes can be determined by setting the R.H.S of Eq. (2-19) to zero (*in vacuo* or homogeneous p.d.e), and solving the p.d.e for a given set of boundary conditions. The natural frequency is calculated by inserting the mode shape solution back into the homogeneous p.d.e and solving for the natural frequency ω_n . For a simply supported beam, the boundary conditions at $x = 0$ and $x = a$ are

$$\begin{aligned} \eta(0, t) &= \eta(a, t) = 0 \\ M_z(0, t) &= \frac{\partial^2 \eta(0, t)}{\partial x^2} = M_z(a, t) = \frac{\partial^2 \eta(a, t)}{\partial x^2} = 0 \end{aligned} \quad (2-21)$$

and the mode shape and natural frequency are

$$\begin{aligned} \phi_n(x) &= \sin\left(\frac{n\pi}{a}x\right) = \sin(k_b x) \\ \omega_n &= \left(\frac{n\pi}{a}\right)^2 \sqrt{\frac{D}{m}} = (k_b)^2 \sqrt{\frac{D}{m}} \\ k_b &= \frac{n\pi}{a} \end{aligned} \quad (2-22)$$

If the beam natural frequencies in Eq. (2-22) were used to model the plate partition, the model would make the plate too flexible. Therefore, a pseudo-plate model is approximated by matching the simply supported plate natural frequencies to that of a simply supported beam. The natural frequencies for a simply supported plate are given by

$$\omega_{nq} = \left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2\right] \sqrt{\frac{D}{m}} \quad (2-23)$$

Setting the natural frequency from Eq. (2-22) equal to that of Eq. (2-23) yields

$$\left(\frac{n\pi}{a_e}\right)^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2 \quad (2-24)$$

where a in Eq. (2-22) has been replaced by an effective length a_e . Setting q in Eq. (2-24) to unity and solving for a_e yields

$$a_e = \frac{abn}{\sqrt{a^2 + b^2 n^2}} \quad (2-25)$$

Therefore the natural frequencies and mode shapes used in this pseudo-plate model are

$$\omega_n = \left(\frac{n\pi}{a_e}\right)^2 \sqrt{\frac{D}{m}} = (k_{be})^2 \sqrt{\frac{D}{m}} \quad (2-26)$$

and

$$\phi_n(x) = \text{Sin}\left(\frac{n\pi}{a_e}x\right) = \text{Sin}(k_{be}x)$$

where k_{be} is now

$$k_{be} = \frac{n\pi}{a_e} \quad (2-27)$$

Inserting Eq. (2-20) into Eq. (2-19), then multiplying by ϕ_r and integrating over the domain from $x=0$ to $x=a$ produces

$$\int_0^a \phi_r D A_n \frac{\partial^4 \phi_n}{\partial x^4} dx - \int_0^a \phi_r \omega^2 m A_n \phi_n dx = \int_0^a \phi_r (P_{in} - P_{out}) b dx \quad (2-28)$$

This equation is called the weak modal formulation of the p.d.e. The modal orthogonality conditions are given by

$$\begin{aligned} \int_0^a m \phi_r \phi_n dx &= \delta_{rn} \\ \int_0^a D \phi_r \frac{\partial^4 \phi_n}{\partial x^4} dx &= \delta_{rn} \omega_r \end{aligned} \quad (2-29)$$

where δ_{rn} is the Kronecker Delta function, which has a value of one when r and n are equal, and zero when they are not equal. Applying the conditions in Eq. (2-29) into Eq. (2-28) produces

$$A_n \omega_n^2 - \omega^2 A_n = (P_{in} - P_{out}) b \int_0^a \phi_n(x) dx = (P_{in} - P_{out}) b \psi_n \quad (2-30)$$

where $\psi_n = \int_0^a \phi_n(x) dx$, and repeated indices in this equation do not indicate summation.

ψ_n has a value of one when n is odd and zero when n is even. Therefore, only symmetric modes transmit acoustic energy from one chamber of the DWNIT to the other. Solving for A_n in Eq. (2-30) yields

$$A_n = \frac{(P_{in} - P_{out}) b \psi_n}{\omega_n^2 - \omega^2} \quad (2-31)$$

From Eqs. (2-31) and (2-20), the dynamic displacement response of the plate is given by

$$\eta(x, t) = (P_{in} - P_{out}) \sum_{n=1}^m \frac{b \psi_n \phi_n(x)}{\omega_n^2 - \omega^2} e^{i\alpha x} \quad (2-32)$$

The plate velocity response is given as

$$\dot{\eta} = \frac{\partial \eta}{\partial t} = i\omega (P_{in} - P_{out}) \sum_{n=1}^m \frac{b \psi_n \phi_n(x)}{\omega_n^2 - \omega^2} e^{i\alpha x} \quad (2-33)$$

To determine the radiated field, the maximum velocity function in the pseudo-plate was calculated using the no-penetration boundary conditions. Consequently, Eq. (2-33) becomes

$$\dot{\eta} = \alpha(P_{in} - P_{out})e^{i\omega t} \quad (2-34)$$

where

$$\alpha = \text{Max}_l \left[i\omega b \sum_{n=1}^m \frac{\psi_n \phi_n(x)}{\omega_n^2 - \omega^2} \right] \quad (2-35)$$

where the Max function returns the complex number with the largest modulus (magnitude). The acoustic pressure field generated by the vibrating plate is given by

$$p_{out} = P_{out} e^{i(\omega t - k_1 x - k_2 y)} \quad (2-36)$$

The no-penetration boundary condition requires that the acoustic particle velocity and the plate velocity must match. Therefore, applying this boundary condition and using Eq. (2-36) yields

$$\dot{\eta}(x,0,t) = -\frac{1}{i\omega\rho} \frac{\partial p_{out}}{\partial y} = \frac{k_2}{\omega\rho} p_{out}(x,0,t) \quad (2-37)$$

k_2 must satisfy the radiation dispersion relationship

$$k^2 = k_1^2 + k_2^2 = \left(\frac{\omega}{c} \right)^2$$

or

$$k_2 = \sqrt{\left(\frac{\omega}{c} \right)^2 - \omega^2 \left(\frac{m}{D} \right)^{\frac{1}{2}}} \quad (2-38)$$

where

$$k_1 = k_b = \sqrt{\omega \left| \frac{m}{D} \right|^{\frac{1}{2}}} \quad (2-39)$$

Using Eqs. (2-34), (2-37), (2-38) and rearranging yields

$$\frac{P_{out}}{P_{in}} = \frac{\alpha}{\left| \alpha + \frac{k_2}{\omega \rho} \right|} \quad (2-40)$$

The intensity transmission coefficient is given by

$$T_{12} = \frac{I_{12out}}{I_{12in}} = \frac{P_{12out} u_{12out}^*}{P_{12in} u_{12in}^*} = \frac{P_{12out} \frac{P_{12out}^*}{Z^*}}{P_{12in} \frac{P_{12in}^*}{Z^*}} = \left| \frac{P_{12out}}{P_{12in}} \right|^2 \quad (2-41)$$

Therefore using Eqs. (2-40) and (2-41), the intensity transmission coefficient is

$$T_{12} = \left| \frac{\alpha}{\left| \alpha + \frac{k_2}{\omega \rho} \right|} \right|^2 \quad (2-42)$$

where k_2 must be greater than zero for acoustic energy to be transmitted through the partition.

2.3 Partition Thickness and Structural Damping Loss Factor

Two important factors that must be considered in the design of the chamber partition, are the amount of structural damping loss factor to be added to the partition and the thickness of the partition plate. Near the sample the partition is tapered to a point (as seen in Fig. 2-4, in order to minimize the effect of the partition pressing on the sample. Intuitively, the thicker the partition is made the greater the SPL drop across the plate and the slope of the taper required. If the slope of the taper is too large, the acoustic field will be disturbed and the reduced acoustic impedance will be the coupling effect between the near field effects of the partition taper and the end sample tested. Therefore, the partition thickness should be designed as thin as possible, while still providing an adequate SPL drop across the partition.

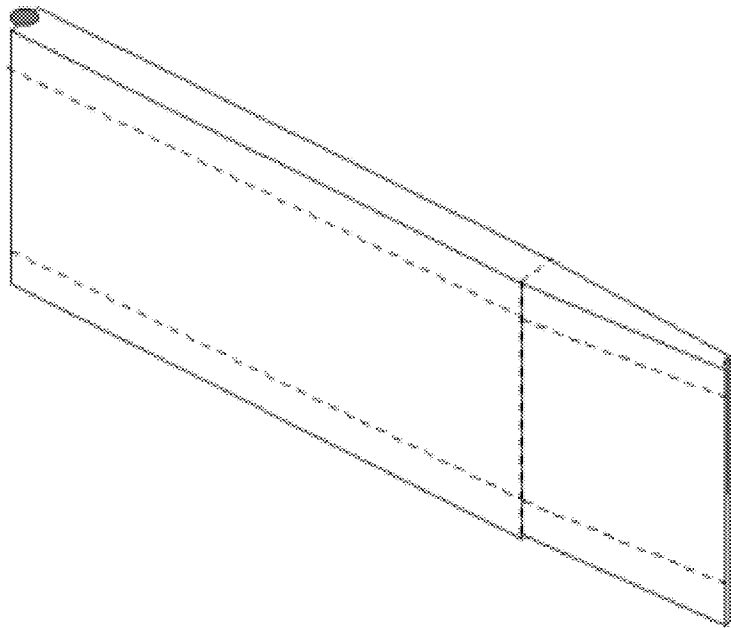


Figure 2-4. End taper design of plate partition.*

* Reprinted with permission of Mr. Tony Parrott.

Intuitively, it can also be seen that by increasing the damping loss factor, the SPL drop will increase, since a greater amount of the total incident energy will be dissipated rather than transmitted through the plate partition. Consequently, the partition could be made thinner by simply adding more damping. Nevertheless, this effect is also undesirable because it decreases the amount of the reflected energy back into the incident energy chamber. This in effect would “soften” the wall, shifting the crosssectional modes from the desired “hard” wall conditions. Moreover, adding significant damping will increase the required acoustic power from the drivers to achieve the equivalent SPLs desired in the chambers. Therefore, the amount of damping should also be minimized.

This problem lends itself to a constrained minimization subject to a set of SPL drop, damping, and plate thickness constraints.³ The goal is to determine the minimum SPL drop (must not be less than 40 dB) that can be achieved under the constraints of

$$\begin{aligned} 0.05 \leq \tau \leq 0.35 \\ 0.01 \leq \gamma \leq 0.1 \end{aligned} \tag{2-43}$$

where the thickness (τ) is given in inches in Eq. (2-43). The loss factor (γ) can be changed by adding commercially available viscoelastic damping polymers.⁴ A MATLAB program was written that searches for the minimum SPL drop in the Δ SPL power spectrum utilizing the models developed in this chapter for the partition.

References

1. Kinsler, L.E., Frey, A.R., Coppens, A.B., and Sanders, J.V., *Fundamental of Acoustics*, John Wiley & Sons, New York, 1982.
2. Fahy, F., *Sound Structural Vibration*, Academic Press, New York, 1998.
- 3 . Arora, J. S., *Introduction To Optimum Design*, McGraw-Hill, New York, 1989.
- 4 . *Viscoelastic Damping Polymers*, 3M Bonding Systems Division, St. Paul, 1999.

3. Results

The initial design of the partition plate was 2 in (5.08cm) high by 16 in (40.64cm) long and 0.25 in (0.635cm) thick. The chosen plate material was steel and the frequency range of interest was 1 to 10 kHz. All the intensity transfer functions can assume values between zero and one. From Chapter 2, an intensity transfer function value of zero indicates that all the incident acoustic energy is reflected, while a value of one indicates that all the acoustic energy is transmitted.

The partition design requirement was to achieve at least 40dB SPL drop (isolation) between the two chambers of the DWNIT. From the energy model developed in Section 2.1, it can intuitively be seen from Eq. (2-17) that when T_{1e} and T_{2e} are small, the SPL drop across the partition is at its minimum. Therefore, these conditions would be close to the worst-case scenario the partition will experience.

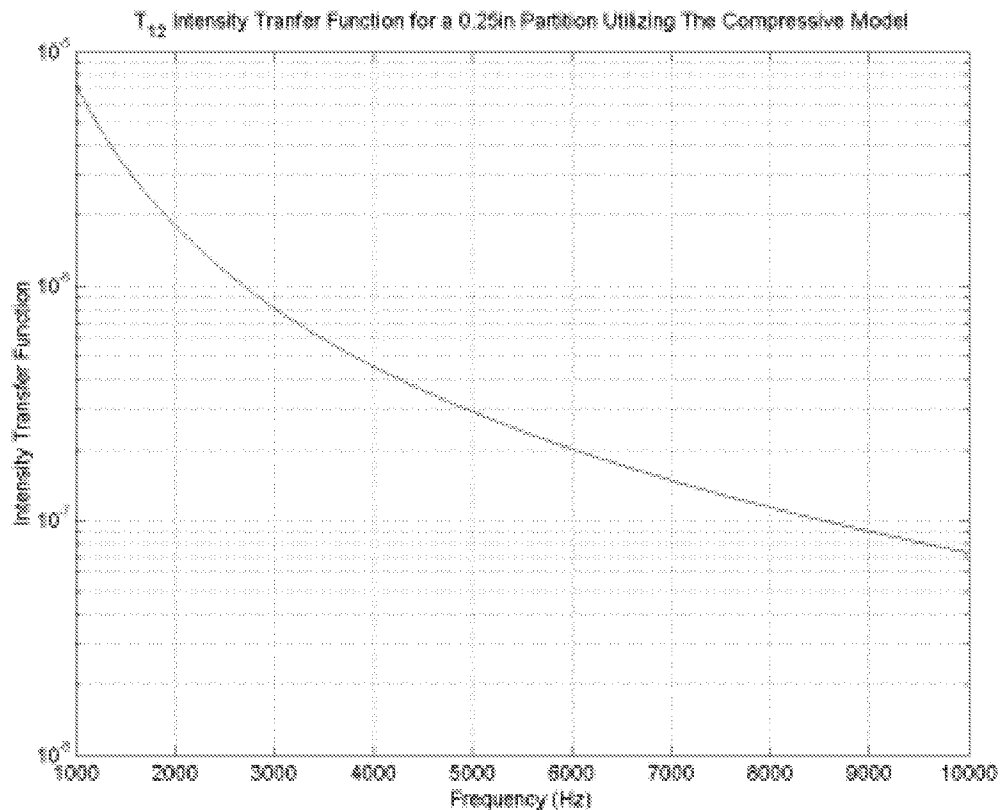


Figure 3-1. Intensity Transfer Function for a 0.25in (0.635cm) thick partition utilizing the Compressive Model.

Figure 3-1 shows the transmission function T_{12} for a 0.25in (0.635cm) thick steel partition utilizing the compressive model. The figure shows a small magnitude for the transmission, indicating a small transfer of energy between the two chambers of the DWNIT. The figure also shows that the transmission decreases with increasing frequency. This well known effect occurs, because as the frequency of the acoustic wave increases, the wavelength decreases. The shorter wavelengths are better able to “see” the plate thickness and therefore a greater percentage of the acoustic energy gets reflected.

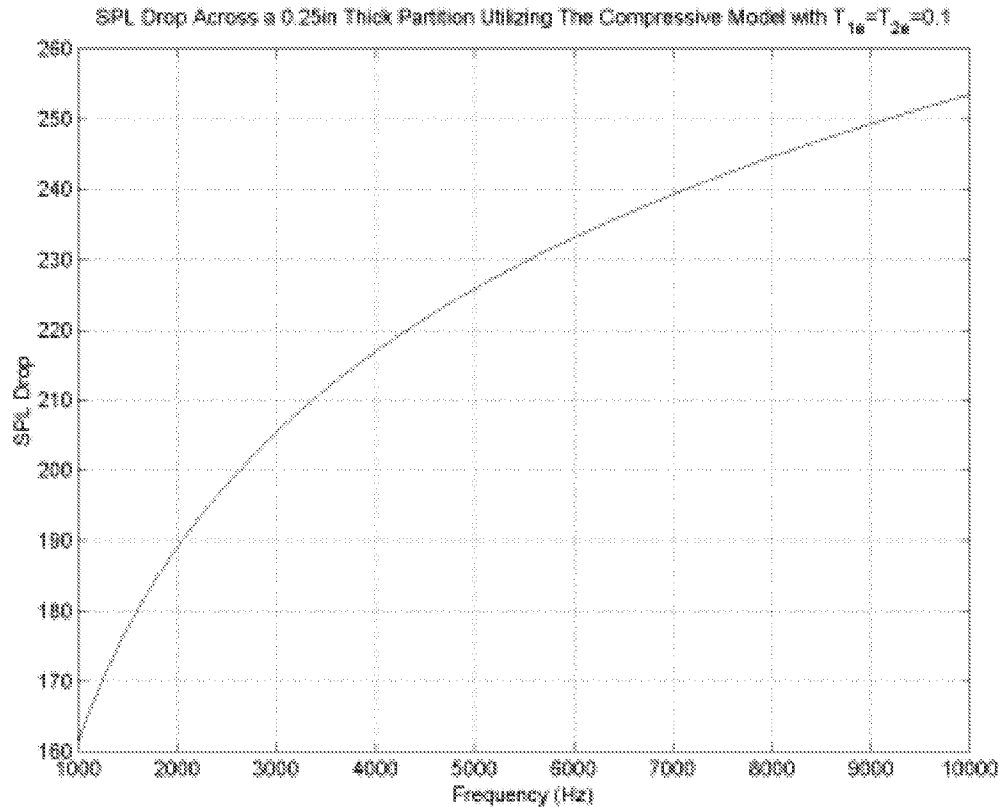


Figure 3-2. SPL drop across a 0.25in (0.635cm) thick partition utilizing the Compressive Model with $T_{1e}=T_{2e}=0.1$.

Figure 3-2 shows the SPL drop across a 0.25in (0.635cm) thick steel partition utilizing the compressive model with $T_{1e}=T_{2e}=0.1$. The figure shows that the minimum SPL drop across the partition is 160dB, well above the 40dB minimum design requirement. The figure shows the SPL drop increasing with frequency, which is due to the partition’s intensity transfer function decrease with frequency.

The results shown in Figs. 3-1 and 3-2 demonstrate that the partition would provide adequate isolation between the chambers of the DWNIT, if the plate partition was infinitely long and only compressive waves were present. The pseudo-plate model developed in Section 2.2.2 will be analyzed next to determine the effects of bending waves on the partition.

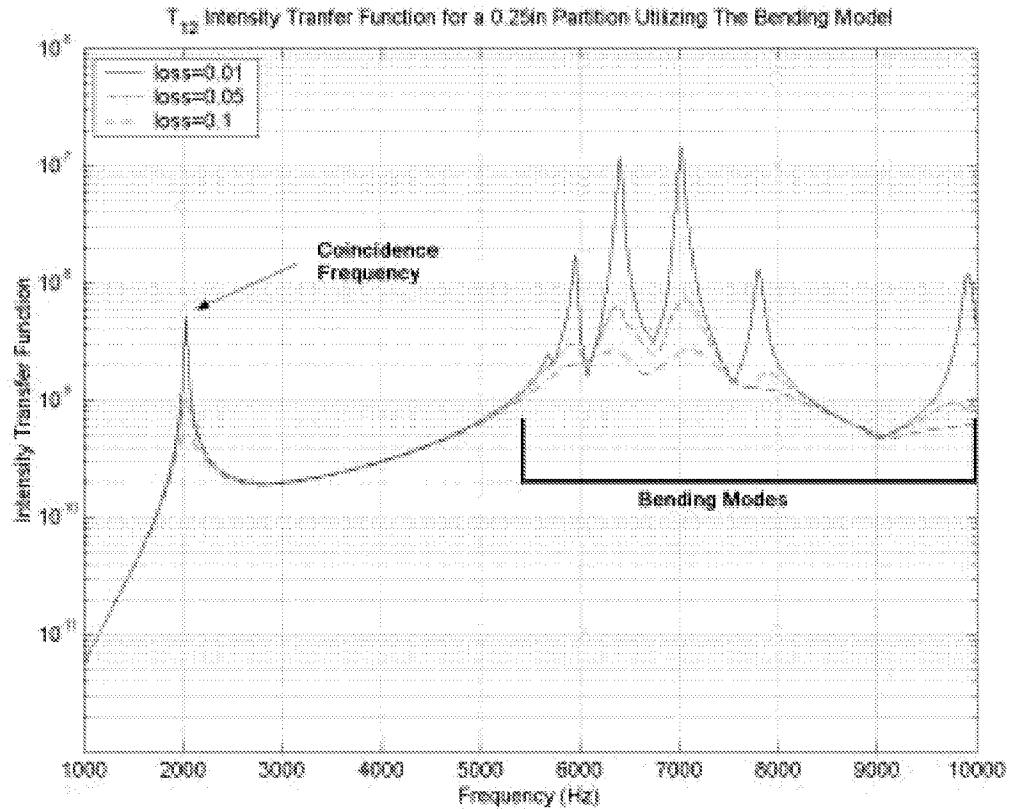


Figure 3-3. Intensity Transfer Function for a 0.25in (0.635cm) thick partition utilizing the Bending Model.

Figure 3-3 shows the intensity transfer function T_{12} for a 0.25in (0.635cm) thick steel partition utilizing the bending model. Figure 3-4 shows the corresponding SPL drop across a 0.25in (0.635cm) thick steel partition with $T_{1e}=T_{2e}=0.1$. These figures are essentially mirror images of each other, where high transmission produces low SPL drop and vice versa. These figures show low levels of the intensity transfer function and high SPL drops across the partition. Figure 3-4 shows that the minimum SPL drop across the

partition is around 245dB with a loss factor of 0.01. This SPL drop is well above the 40dB minimum design requirement.

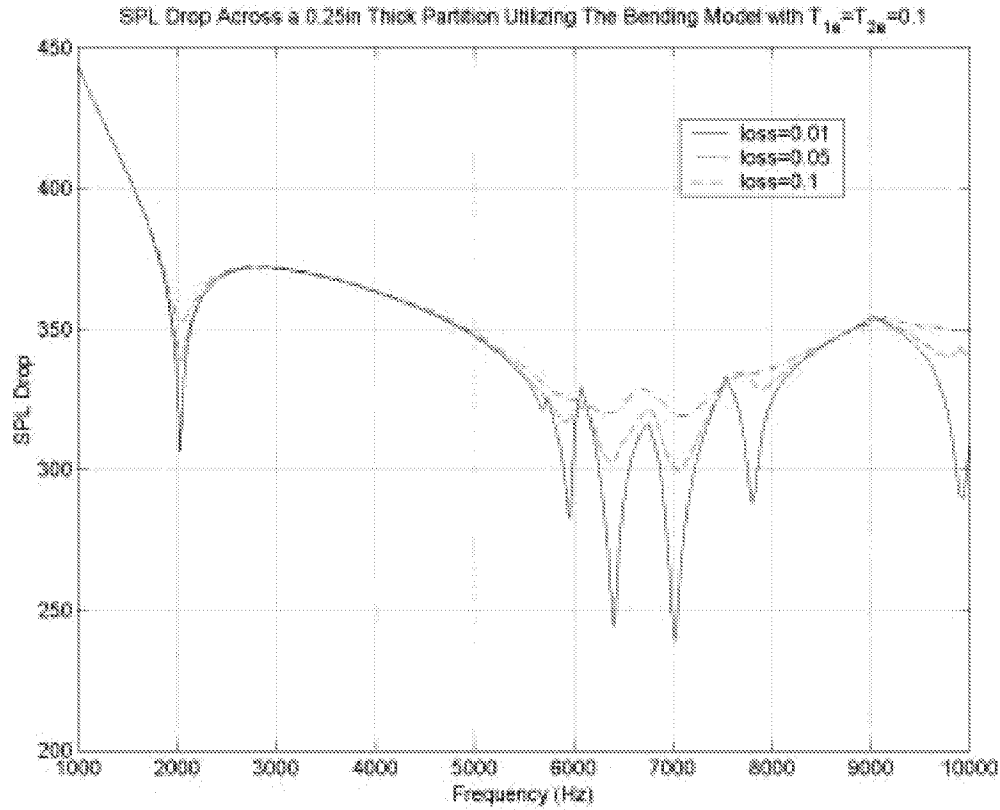


Figure 3-4. SPL drop across a 0.25in (0.635cm) thick partition utilizing the Bending Model with $T_{1e}=T_{2e}=0.1$.

The first peak in the Fig. 3-3 and dip in Fig. 3-4 is the coincidence or critical frequency. This frequency occurs when the bending wave number and the incident wave number along the length of the plate match, as seen in Fig. 3-5. In essence at this point, the projected axial acoustic wavelength matches that of the plate, thereby making the plate transparent to the incident acoustic wave. Therefore, without damping at this frequency, all the incident acoustic waves would get transmitted. This would cause an increase in the intensity transmission and a reduction in SPL drop across the partition, as seen in Figs. 3-3 and 3-4. Since k_2 is essentially imaginary below (for no damping) this critical frequency, there is no acoustic transmission across the plate until the coincidence

frequency is reached. From Figs. 3-3, 3-4, and 3-5, the coincidence frequency for the 0.25 in steel partition plate is around 2000Hz.

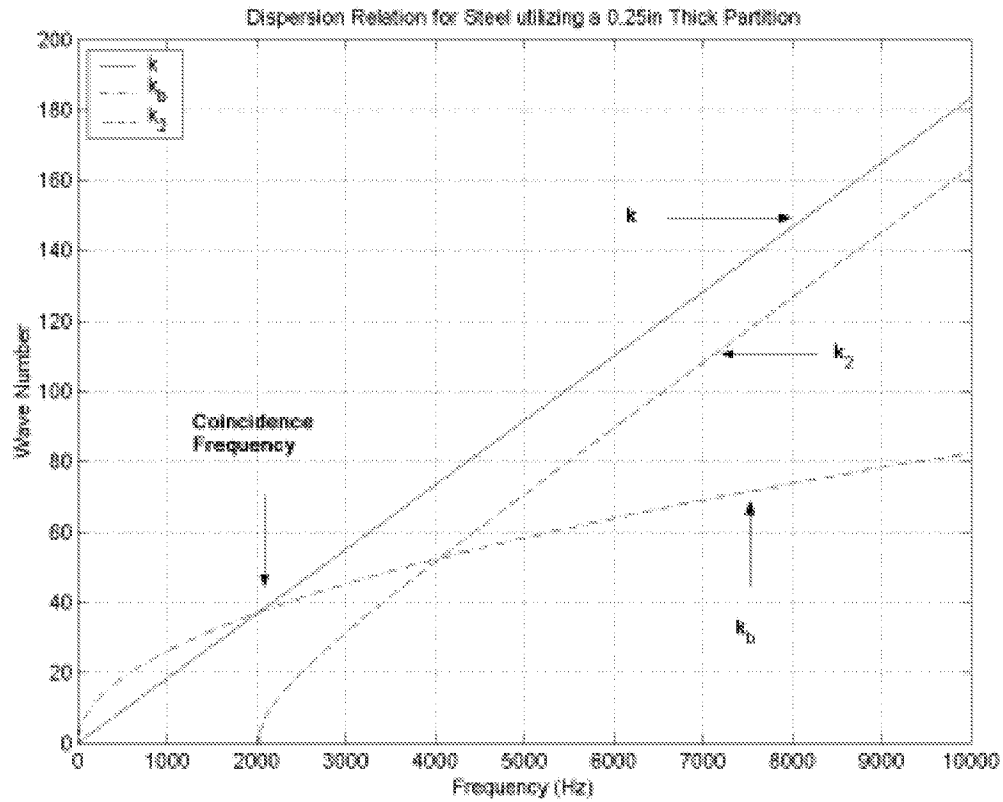


Figure 3-5. Dispersion relationship utilizing a 0.25in (0.635cm) thick partition.

The second set of peaks and dips in Figs. 3-3 and 3-4, respectively, are the partition's bending modes. The first bending mode occurs just under 6000Hz. The figure shows that a moderate increase in damping loss from 0.01 to 0.05 can significantly increase this minimum SPL drop from 245dB to 300dB. This increase in damping can be achieved by using commercially available viscoelastic damping polymers.

Figure 3-4 showed that 0.25in (0.635cm) thick steel partition would provide at least 40dB isolation between the two chambers of the DWNIT for a range of loss factors between 0.01 and 0.1. The next step in the design process was to determine how thin the partition could be made, and how much structural damping loss factor would be required to comply with the minimum allowable SPL drop design requirements.

Figure 3-6 shows the lowest SPL drop over the 1 to 10kHz frequency range, for plate thickness of 0.05in to 0.35in and loss factors between 0.01 and 0.1, utilizing the pseudo-plate model. Figure 3-7 shows the corresponding compressive model results. Figure 3-6 also shows that adding a small amount of damping greatly increases the performance of the partition, and that adding more than this level does not greatly improve the partition's performance.

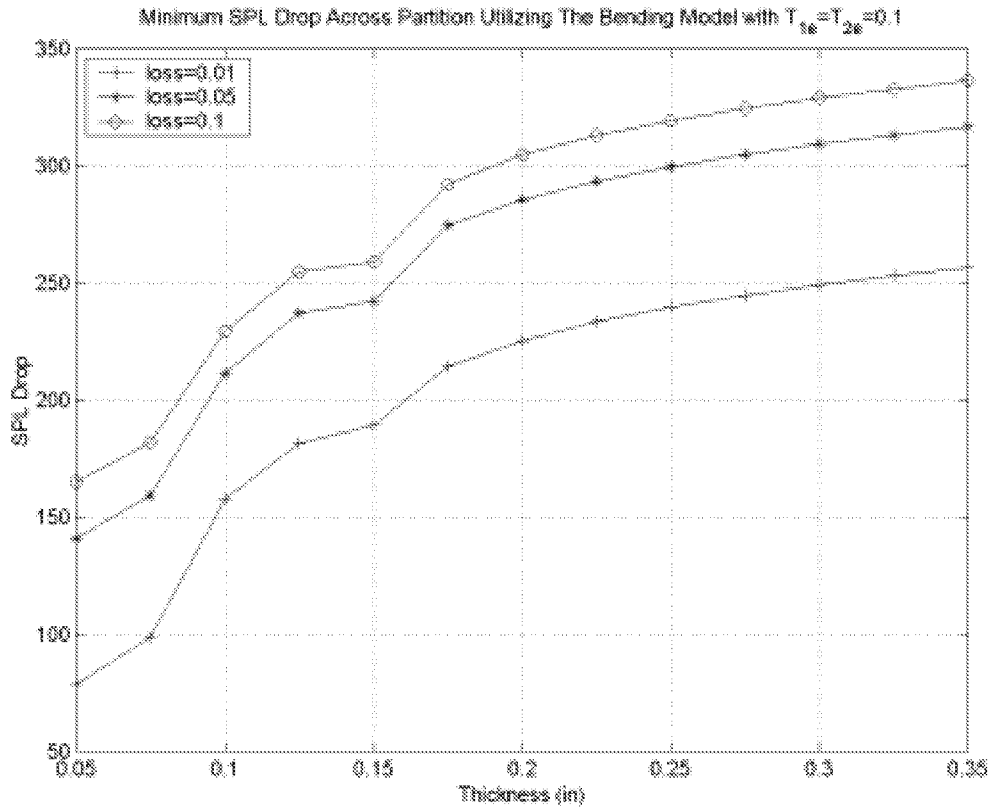


Figure 3-6. Minimum SPL Drop vs. Partition Thickness Utilizing the Bending Model.

Figures 3-6 and 3-7 show the graphical solution of the constrained minimization in Eq. (2-43) using the bending and compressive models, respectively. Both plots show that, for this range of thickness and damping values, all SPL drop values are above the 40dB design requirement. From the constrained minimization perspective, the thickness constraint in Eq.(2-43) is binding. Therefore, from the plot, the solution to Eq. (2-43) is $\text{dB_Drop}(\tau, \gamma) = 75\text{dB}$ (Bending), 97dB (Compressive), where $\tau = 0.05\text{in}$ and $\gamma = 0.01$.

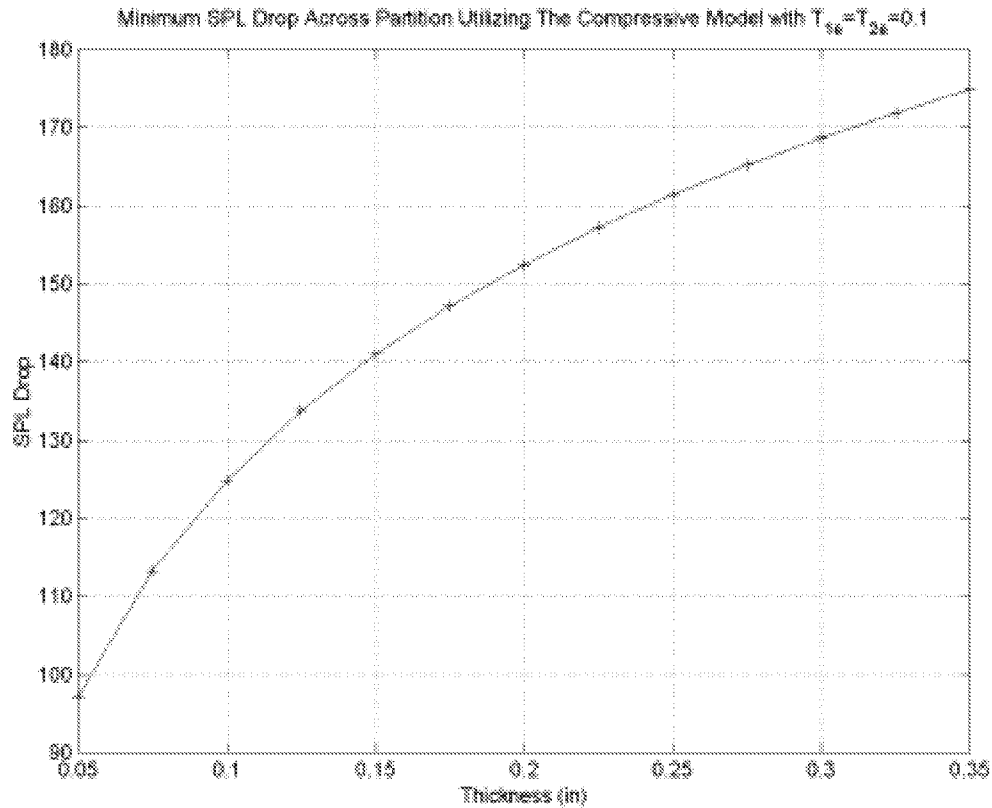


Figure 3-7. Minimum SPL Drop vs. Partition Thickness Utilizing the Compressive Model.

Although the plate thickness and damping ranges in these figures satisfy the design requirements, simply selecting the solution to Eq. (2-43) might not be recommendable. Both Figs. 3-6 and 3-7 show that, as the partition thickness decreases, the SPL drops more rapidly across the plate. Therefore, small variations in thickness and damping due to manufacturing and installation process might put the partition design out of tolerance. Possible reasonable values of damping and thickness are 0.05 and 0.1in (0.127 and 0.254cm), respectively. This damping/thickness combination is high enough in SPL drop and shallow enough in terms of the slope of this curve to provide a “safe” design.

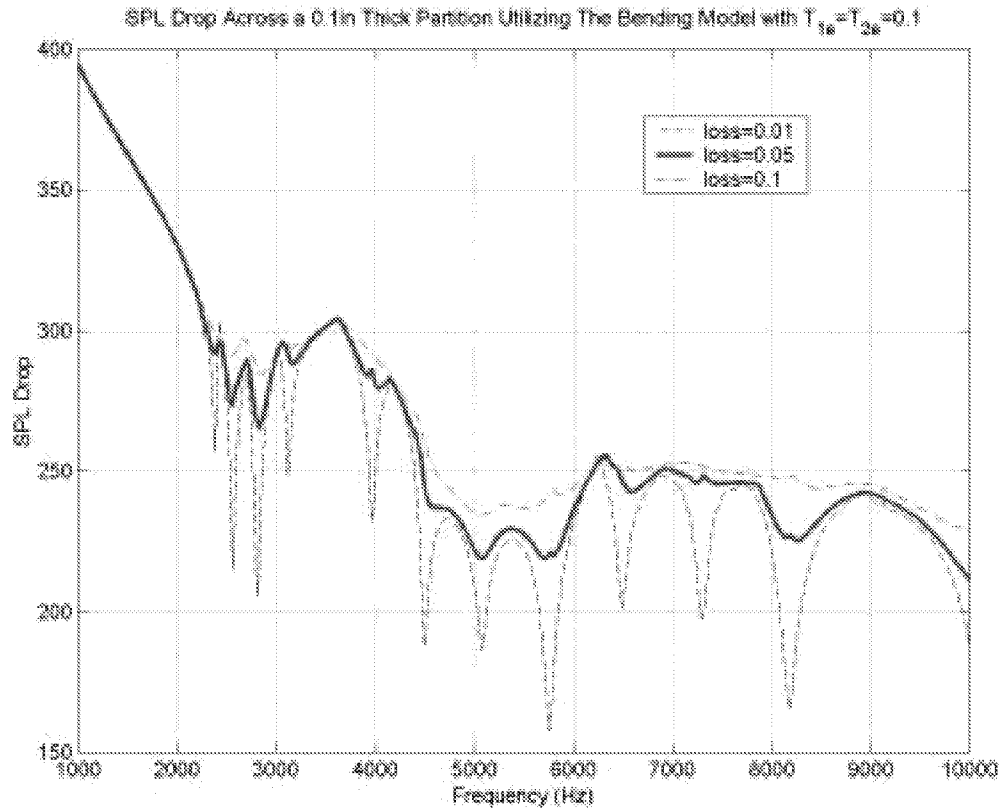


Figure 3-8. SPL drop across a 0.1in (0.254cm) thick partition utilizing the Bending Model with $T_{1e}=T_{2e}=0.1$.

Figure 3-8 shows the SPL drop across a 0.1in (0.254cm) steel plate partition utilizing the bending model. As expected, the plot shows that there are more bending modes present with a 0.1in (0.254cm) thick plate than with the original 0.25in (0.635cm) plate thickness design. Nevertheless, using a 0.05 loss factor damps out most of the mode dips. The lowest SPL drop under this design is about 225dB.

It is important to note that the design analysis so far has been conducted assuming that the end intensity transfer functions T_{1e} and T_{2e} are equal to 0.1. In other words, the end samples are reflective and their absorptivity is small. Although this is a good limiting assumption for the design, in reality normal incidence tubes are used to test very absorptive materials that would attenuate sound in noise control applications. For applications at NASA Langley's NIT, the end sample impedances rarely exceed $5\rho c$. The intensity transfer function can be related to the impedance by

$$T = \frac{4\theta}{(\theta+1)^2 + \chi^2} \quad (3-1)$$

where θ and χ are the nondimensional resistance and reactance. Letting θ equal 5 and χ equal zero in Eq. (3-1) yields $T=0.555$. Figures 3-9 and 3-10 show the SPL drop across the 0.1in (0.254cm) partition with $T_{1e}=T_{2e}=0.555$ using the bending and compressive model, respectively. From the figures, the minimum SPL drop across the partition is 250dB and 160dB for the bending and compressive models, respectively.

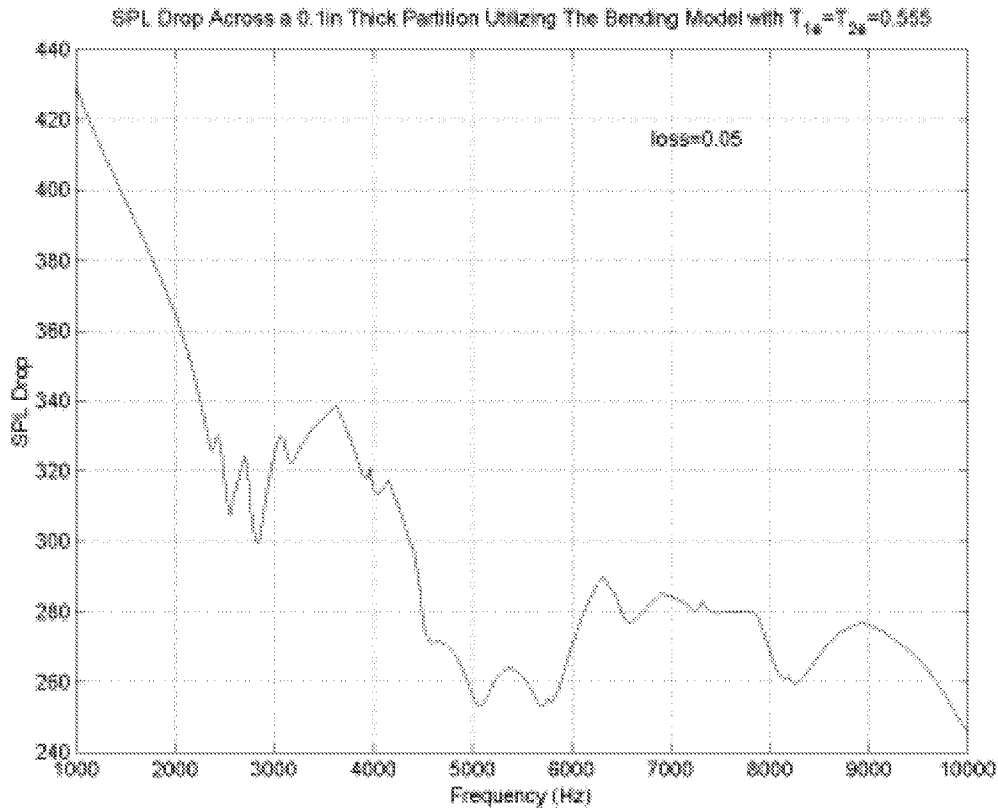


Figure 3-9. SPL drop across a 0.1in (0.254cm) thick partition utilizing the Bending Model with $T_{1e}=T_{2e}=0.555$.

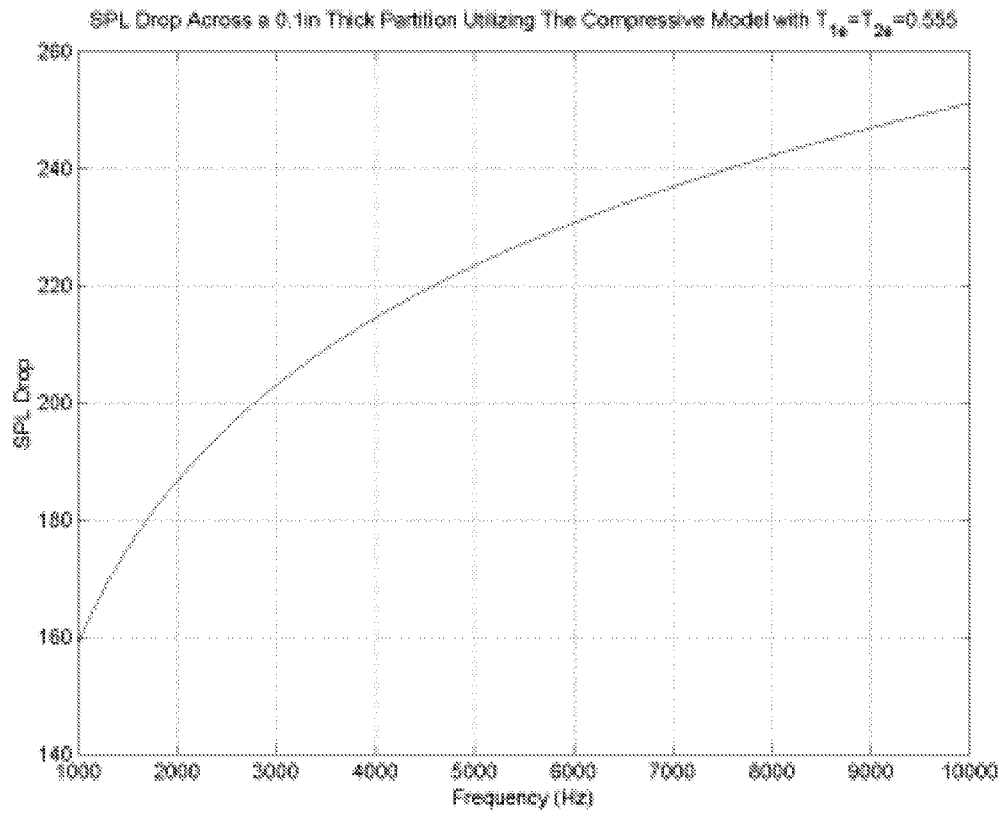


Figure 3-10. SPL drop across a 0.1in (0.254cm) thick partition utilizing the Compressive Model with $T_{1e}=T_{2e}=0.555$.

4. Conclusions

An energy model was developed of the two chambers of the DWNIT. This energy model determined the SPL drop across the two chambers assuming an intensity transfer function for the chamber's partition. The model allowed the end sample intensity transfer functions to vary to determine their effect on the SPL drop across the partition. To model the partition, two intensity transfer function models were studied. The first intensity transfer function model used was a simple compressive model and the other was a pseudo-plate bending model.

The initial partition design (2" high, 16" long, 0.25" thick) was predicted to provide at least 160 dB SPL drop across the partition with the compressive model, and at least 240 dB SPL drop with the bending model. These results were produced when the structural damping loss in the bending model was 0.01 and the model end chamber sample transmissions were set to 0.1. This was done to analyze the partition under a close to worst-case scenario.

Since these results predicted more SPL drop than required, a plate thickness optimization algorithm was developed. The results of the algorithm routine indicated that a plate with the same height and length, but with a thickness of 0.1" and 0.05 structural damping loss, would provide an adequate SPL isolation between the chambers. This partition design would provide 210 dB and 125 dB SPL drop across the partition, utilizing the compressive and bending model, respectively. These results were also computed with the end chamber sample transmissions set to 0.1. When more realistic end intensity transmission coefficients (e.g., 0.555) were used, the compressive and bending model predicted a partition isolation of 250dB and 160dB, respectively.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE April 2002	3. REPORT TYPE AND DATES COVERED Contractor Report		
4. TITLE AND SUBTITLE Design of a Dual Waveguide Normal Incidence Tube (DWNIT) Utilizing Energy and Modal Methods		5. FUNDING NUMBERS NAS1-00135, Task Order 02RBF 781-30-14-01		
6. AUTHOR(S) Juan F. Betts				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Lockheed Martin Corporation Hampton, VA 23681		8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Langley Research Center Hampton, VA 23681-2199		10. SPONSORING/MONITORING AGENCY REPORT NUMBER NASA/CR-2002-211635		
11. SUPPLEMENTARY NOTES Langley Technical Monitor; Michael G. Jones				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified-Unlimited Subject Category 71 Availability: NASA CASI (301) 621-0390		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words) This report investigates the partition design of the proposed Dual Waveguide Normal Incidence Tube (DWNIT). Some advantages provided by the DWNIT are (1) Assessment of coupling relationships between resonators in close proximity, (2) Evaluation of "smart liners", (3) Experimental validation for parallel element models, and (4) Investigation of effects of simulated angles of incidence of acoustic waves. Energy models of the two chambers were developed to determine the SPL drop across the two chambers, through the use of an intensity transmission function for the chamber's partition. The models allowed the chamber's lengthwise end samples to vary. The initial partition design (2" high, 16" long, 0.25" thick) was predicted to provide at least 160 dB SPL drop across the partition with a compressive model, and at least 240 dB SPL drop with a bending model using a damping loss factor of 0.01. The end chamber sample transmissions coefficients were set to 0.1. Since these results predicted more SPL drop than required, a plate thickness optimization algorithm was developed. The results of the algorithm routine indicated that a plate with the same height and length, but with a thickness of 0.1" and 0.05 structural damping loss, would provide an adequate SPL isolation between the chambers.				
14. SUBJECT TERMS Energy model, compressive model, bending model, SPL drop, structural damping			15. NUMBER OF PAGES 40	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL	